ary R_o, where the particle is considered captured, and over the length H of the cyclone, the grade efficiency of the cyclone can be established as

$$\ln\left(\frac{R_o}{r}\right) = \frac{1}{2}\ln\left(\frac{1}{1-\eta}\right) = \left(\frac{\rho d^2}{18\mu}\right)(2\pi S)^2 \left[\frac{Q_p + \frac{1}{2}Q_s}{\pi R_o^2 H}\right] - \frac{1}{2}\ln\left(1 + \frac{Q_s}{Q_p}\right) \quad (16)$$

or

$$d_\eta = \left(\frac{3}{2\pi s}\right)\sqrt{\left(\frac{\mu}{\rho_p}\right)(\tau)\ln\left[\left(\frac{1}{1-\eta}\right)\left(1 + \frac{Q_s}{Q_p}\right)\right]} \quad (17)$$

where τ is the residence time in the cyclone:

$$\tau = \frac{\pi R_o^2 H}{Q_p + \frac{1}{2}Q_s} \quad (18)$$

It is interesting to note that the inclusion of the radial gas velocity term leads to the conclusion that the cyclone will only capture particles above a certain minimum size given by

$$d_o = \left(\frac{3}{2\pi s}\right)\sqrt{\left(\frac{\mu}{\rho_p}\right)(\tau)\ln\left[1 + \frac{Q_s}{Q_p}\right]} \quad (19)$$

This conclusion was previously arrived at by balancing the drag and maximum centrifugal force at the core boundary which yielded a conservative estimate of the minimum particle size. Figure 3 indicates the grade efficiencies determined by the previous method, the present method, and experimentally for a 50 ft^3/min unit. As the curve indicates, the inclusion of the radial gas velocity impedes particle collection, and the entire grade efficiency curve is shifted towards the experimental curve. The difference between the experimental results and the present model is most likely explained by the fact that the assumption of uniform flow of the secondary gas into the core is only an approximation. Turbulence, and its effects on particle reentrainment and capture, cannot be accounted for in this model, and this too may be an important cause for the discrepancy between experimental and calculated results.

A final observation to be made is that the model predicts different sized cyclones can maintain similar grade efficiencies for a given gas-particle system by maintaining

$$\frac{\sqrt{\tau}}{s} = \text{constant}$$

and

$$\frac{Q_s}{Q_p} = \text{constant}$$

NOTATION

d = particle diameter
H = length of cyclone body
Q_p = primary gas flow rate
Q_s = secondary gas flow rate
R_o = radius of rotational flow core
r = radius
S = number of revolutions of gas flow in core
t = time
V_z = gas axial velocity
V_r = gas radial velocity
z = axial position

Greek Letters

η = collection efficiency
μ = gas viscosity
ρ_p = particle density
τ = residence time in the core
ω = angular velocity

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Velocity Measurements in Two-Phase Bubble-Flow Regime with Laser-Doppler Anemometry

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Several analytical and experimental studies do exist in the area of two-phase flow involving a gas and a Newtonian liquid; in spite of these, it is still recognized that two-phase studies remain a complex area where the progress has been rather slow. Mahalingam and Valle (1972) and Mahalingam (1975) point out the existence of discrepancies between analytical and experimental results in two-phase, gas-liquid flow. In order to provide an improved interpretation of experimentally observed two-phase flow phenomena through analytical models, it is necessary a priori to develop novel experimental techniques to generate reliable data. The techniques described here provide for an accurate measurement of bubble rise velocity in a stagnant fluid and of the velocity of individual phase (and hence the slip velocity) in two-phase gas-liquid systems, in the bubble-flow regime. The laser-Doppler anemometer (LDA) is used in these measurements, thus confirming the potential of LDA systems in two-phase flow measurements.

The laser-Doppler anemometer has, over the last few years, rapidly evolved as a standard and absolute technique for measurement of velocities of single-phase systems such as liquids or gases. Doppler phenomenon is readily observed, where two laser beams are brought to a

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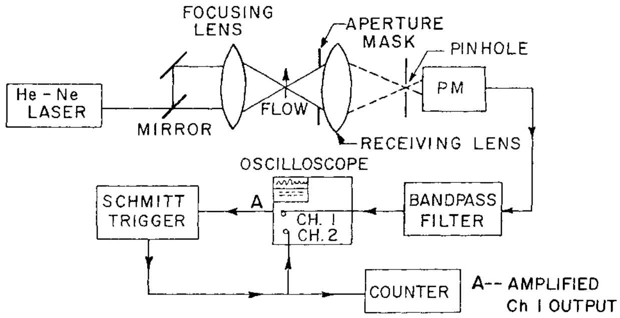

Fig. 1. Optical arrangement and signal processing system.

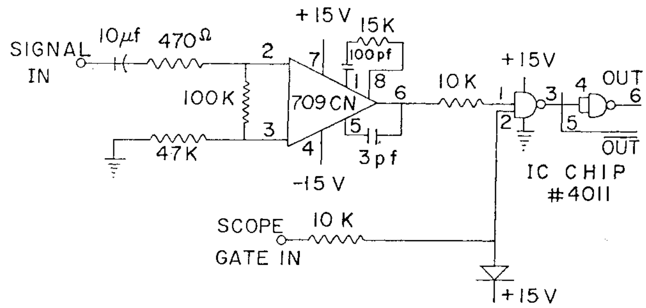

Fig. 2. Schmitt trigger circuit.

focus at a single point in the flow field and a particle is allowed to traverse this focal region. In view of the coherence characteristics of the laser source, the beams will interfere constructively and destructively to establish a set of closely spaced planar interference fringes in the focal region. As a scattering particle traverses the focal volume at a velocity v, the fringes are cut at a rate of

$$\frac{1}{f_D} = \lambda/2v \sin \beta \qquad (1)$$

Interest is currently being focused on the application of LDA to two-phase, gas-liquid flow. Durst and Zare (1975) have attempted some theoretical considerations on light scattering which could be used as a basis for velocity measurements in two-phase flow. For a transparent moving particle, for example an air bubble in water, the formula can be modified to

$$f_D = \frac{2v(\sin \beta - \sin \psi)}{\lambda} \qquad (2)$$

For large L/R ratios and small beam crossing angle β, this equation tends to the universal equation of laser-Doppler anemometry in single-phase flow measurements and is used as such in the present work.

EXPERIMENTAL

Optical Arrangement

A number of optical arrangements including symmetric heterodyne, differential heterodyne, and local oscillator heterodyne are equally effective for single-phase flow measurements. However, in two phase flow studies, the quality of signals and the accuracy of results become a function of optical arrangement. Ben-Yosef et al. (1975) have utilized a symmetric heterodyne Doppler velocimeter, with possible difficulties in measurements for bubbles greater than 1 cm in diameter and also for turbulent flow.

Davies (1973) and Davies and Unger (1973) use two detectors in the local oscillator heterodyne arrangement. The first detector measures the liquid velocity in the forward scatter mode, while the other, in the back scatter mode, measures the bubble size. This is done by adjusting the scope sensitivity to respond only to the scattering signals above some predetermined size. Situations where the flow medium becomes quite turbid and the void fraction is greater than 50% have also been investigated. For taking into account multiple bubbles passing through the plane perpendicular to the flow axis, they adjust the angle β of intersection between the two equal intensity incident beams sufficiently small to allow for overlap of the two beams throughout the tube diameter.

Durst and Zare (1975) have carried out experiments on bubble rise in a stagnant fluid, using an LDA system involving, a double photodiode with 2 mm separation between the active elements, the signals from the single photodiode being used to analyze the velocity of the rising bubble, while the phase differnce between signals from the two photodiodes is being used to measure the radius of curvature of the reflecting surface and, hence, the size of the bubble.

Based on some calculations and experiments, Lading (1971) has observed the configuration which provides the optimum signal-to-noise ratio to be when the two beams are of equal intensity and the direction of detection is along the extension of the bisector of two incident beams.

In our experiments involving two-phase flow in the bubble-flow regime, we have utilized the differential heterodyne arrangement described by Wang (1972), while Figure 1, shown, includes the signal processing system also. The arrangement uses a 15 mW He-Ne laser and a TSI 900 transmitting and receiving optics. The light due to another coherent beam incident from a different direction but scattered from the same scattering region is used as a reference signal.

Signal Processing

The output of the photomultiplier is connected to a bandpass filter to remove the noise in the signal. The trigger level on a Tektronix Dual Beam 7633 Storage Oscilloscope is adjusted so as to display only the high amplitude desirable signal. The signal is amplified about ten times and then fed to a Schmitt trigger and gate circuit to convert it into a square wave; both the signal and the square wave are then observed on the scope simultaneously, using the chop mode. The gate mechanism helps in observing the two signals in single sweep store mode on the scope. The output is then fed to the counter which gives the average frequency or average time period. Undesirable signals are usually eliminated by visual observations at the scope. The Schmitt trigger circuit is given in Figure 2. This circuit is found to work satisfactorily up to 170 KHz. It is inexpensive, permits sweep by sweep operation, and allows accurate average Doppler frequency measurements. A schematic of the Doppler processing system is given in Figure 3.

Two-Phase Flow Measurements

A constant head tank circulating flow arrangement is used, with the fluid(s) flowing vertically upward in the test section, 1.05 in. I.D. The test section is held rigid, while the laser and optical systems are mounted on a milling machine bed to facilitate a three-dimensional traverse. Neither are extensive measurements reported for two-phase flow (bubble-flow regime), nor do recommendations on optimum optical systems for such measurements exist. Either two photomultipliers or two photodiodes, with a complicated signal processing system, have been used in previous work. In the present work, a single photomultiplier is used to measure the velocity of the liquid and that of the gas bubble at a given location; it is observed that the quality of Doppler signals produced by the rising gas bubbles is adequate for analyzing the velocity of rising bubbles.

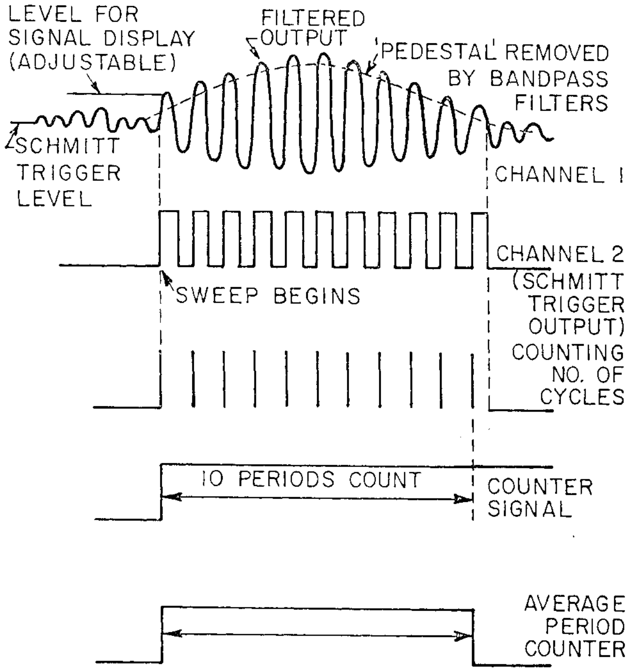

Fig. 3. Schematic of Doppler processing system.

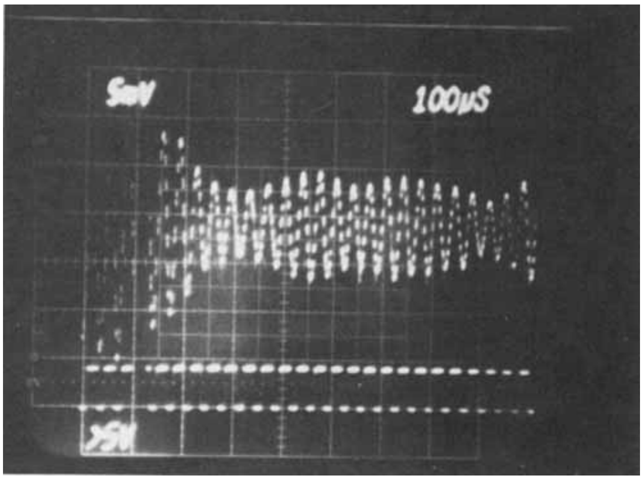

Fig. 4. Signal and square wave—two-phase upward flow, bubble flow regime.

Initial measurements are performed on a single bubble rising in a stagnant fluid. A bubble of uniform size is produced with a bubble generator using air at a regulated pressure and allowed to rise at a constant velocity. When the bubble hits the focal volume, a sweep and a Schmitt trigger display are seen on the scope by adjusting the level position and storage speed. In the case of a moving fluid, there is a continuous signal from the contaminants. In order to isolate the desired signals, the band-pass filter is set by knowing the approximate velocities of the bubble and of the flowing liquid.

The single-bubble technique used above is extended to velocity measurements of each of the phases (and hence the slip velocity) in two-phase flow involving clusters of small gas bubbles moving along with the liquid (Figure 4). A wide range of sizes and velocities of the rising bubbles is obtained by varying the bubble generator and inlet gas pressure. A uniform size distribution and hence a higher accuracy in results is obtained when the gas pressure is low. Higher pressures give rise to a larger number of bubbles, resulting in bubble coalescence and hence in reduced accuracy.

The results obtained with the LDA are verified through multiple-exposure photography. The time interval between exposures is measured accurately on the oscilloscope. For a triple flash, each moving bubble produces three images on the photographic plate. The distance traveled by the bubble during the exposure time interval is directly measured from these photographs (Figure 5). The errors in the bubble size due to the glass tube curvature, the liquid

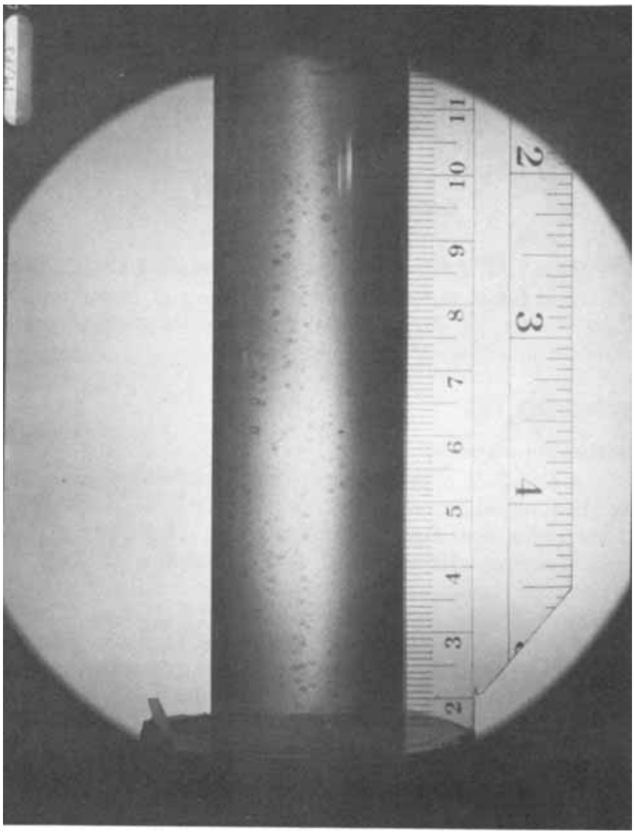

Fig. 5. Triple exposure—two-phase upward flow, bubble flow regime.

Table 1. Comparison of Gas Velocities by Three Different Methods

	Description		Absolute gas velocity, cm/s			Bubble diam., cm	Error in correlation	
Run	Gas	Liquid	LDA	Photo-graphic	Correlation (Table 2)	Photo-graphic	w.r.t. LDA, %	w.r.t. photo-graphic, %
1	Cluster of bubbles	Concurrent upward flow	15.50	15.55	13.52	0.047	12.77	13.05
2	Cluster of bubbles	Concurrent upward flow	20.37	21.24	22.59	0.130	10.89	6.35
3	Single bubble	Concurrent upward flow	23.07	23.04	26.80	0.160	16.16	16.31
4	Single bubble	Concurrent upward flow	21.51	23.21	24.67	0.150	14.69	6.29
5	Single bubble	Stagnant	20.80	21.09	24.64	0.420	18.46	16.83

TABLE 2. TERMINAL VELOCITY OF SINGLE GAS BUBBLES IN LIQUIDS (PEEBLES AND GARBER, 1953)

	Terminal velocity	Range of applicability
Region 1	$v = \dfrac{2R_b^2(\rho_f - \rho_g)\,g}{9\mu_f}$	$Re_b < 2$
Region 2	$v = 0.33g^{0.76}\left(\dfrac{\rho_f}{\mu_f}\right)^{0.52} R_b^{1.28}$	$2 < Re_b < 4.02G_1^{-2.214}$
Region 3	$v = 1.35\left(\dfrac{\sigma}{\rho_f R_b}\right)^{0.50}$	$4.02G_1^{-0.214} < Re_b < 3.10G_1^{-0.25}$ or $16.32G_1^{0.144} < G_2 < 5.75$
Region 4	$v = 1.18\left(\dfrac{g\sigma}{\rho_f}\right)^{0.25}$	$3.10G_1^{-0.25} < Re_b$ $5.75 < G_2$

where

$$Re_b = \frac{2\rho_f v R_b}{\mu_f}$$

$$G_1 = \frac{g\mu_f^4}{\rho_f\sigma^3}$$

$$G_2 = \frac{gR_b^4 v^4 \rho_f^3}{\sigma^3}$$

lens, and the measuring scale seem to cancel each other out, as verified by photographic measurements on particles of known size. The results obtained by the LDA, and photographic methods are then compared in Table 1 with empirical equations available in the literature (Table 2) (Peebles and Garber, 1953).

CONCLUSIONS

The techniques described here show that the LDA system can be successfully utilized in velocity measurements of the individual phases in two-phase gas-liquid flow, in the bubble-flow regime. Several of the analyses hitherto have assumed a no-slip condition in two-phase flow in view of the difficulty in measurement of the slip velocity. The results here show that the slip velocity could indeed be measured without actually disturbing the flow. It is also observed that the errors in the measurements increase with increases in bubble size. Further refinement in the techniques should bring about improvements in bubble size measurements also.

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NOTATION

f_D = Doppler frequency
g = acceleration due to gravity
n = refractive index
r, R = bubble radius
Re = Reynolds number
v = velocity

Greek Letters

β = one half of beam crossing angle
λ = wavelength of laser light
μ = viscosity
ψ = one half angle between two interfering light rays
ρ = density
σ = surface tension

Subscripts

D = Doppler
f = fluid
g = gas

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